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Remarks on the Hadronic Effect in Muon $g-2$: Low Energy Behavior of V^0-π^+ Scattering

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Abstract

The behavior of the V^0-π^+ scattering amplitude (where $V^0 = \rho^0, \omega$, or ϕ) in the low pion momentum limit is studied motivated by its relevance to the theory of muon $g-2$. Current algebra analysis shows that its S-wave component must vanish in the chiral limit under general physical assumptions. We confirm this result with various low energy models of vector meson. Our result justifies the calculation of the charged pion loop part of the hadronic light-by-light scattering contribution to muon $g-2$ within the framework of hidden local symmetry model of low energy hadron dynamics.

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The $\mathcal{O}(\alpha^3)$ hadronic contribution to the anomalous magnetic moment of the muon contains the hadronic light-by-light scattering effect as shown in Fig. 1. Evaluation of such a contribution is currently restricted to purely theoretical analysis. Ref. [1] examined this contribution from the point of view of chiral dynamics. To answer some questions raised about this calculation [2], it was recently re-analyzed in Ref. [3–5]. The study of Ref. [5] tried particularly to respond to the claim that the naive vector meson dominance (VMD) model used in Ref. [1] does not maintain the Ward identity inferred from electromagnetic symmetry [2]. Since this may affect the estimate of charged pion loop contribution to muon $g-2$, Ref. [5] attempted to rectify it using the hidden local symmetry (HLS) approach [6], which incorporates the vector meson explicitly in the effective low energy theory. It was found that the $\rho^0\rho^0\pi^+\pi^-$ vertex is absent in the chiral limit in the HLS approach, contradicting the naive VMD model. The resulting effective Lagrangian enabled us to recover the relevant Ward identity at the same time.

However the first version of Ref. [4] proposed a different effective Lagrangian (eq. (5.6) in Ref. [4]) realizing VMD, which contains the $\rho^0\rho^0\pi^+\pi^-$ vertex in the chiral limit justifying the naive VMD model of Ref. [1]. In the final version of Ref. [4] more complicated form of Lagrangian has been written down which consists of a few sets of several infinite series of higher derivative terms expanded in some mass scale M_X. It gives a familiar VMD structure with this mass scale for each the hadronic light-by-light scattering amplitude. The part of it relevant for the muon anomaly is that found in naive VMD model. Thus *when this scale is set to the rho-meson mass* their numerical result of the charged pion contribution to muon $g-2$ is actually close to the previous result [1] rather than that based on the HLS model [5].

When we consider the ρ^0-π^+ scattering amplitude, its leading behavior in low momentum limit is quite different for theories without or with the $\rho^0\rho^0\pi^+\pi^-$ vertex: The former vanishes at threshold while the latter does not. It would be reasonable to expect that such a difference can be studied starting from a chirally invariant Lagrangian in view of our experiences in the S-wave nucleon-pion scattering, pion-pion scattering, and KSRF relation [7]. In other

words, at least the exponent of momentum in the low momentum limit would not depend on how the vector meson is incorporated in the theory. As was mentioned above, however, the effective Lagrangian of Ref. [5] based on the hidden local symmetry approach and that of Ref. [4] based on the extended Nambu-Jona-Lasinio model give different low energy behaviors.

The purpose of this paper is to settle this issue starting from the chiral symmetry alone without relying on any specific effective Lagrangian. Namely, we investigate the leading low energy behavior of ρ^0-π^+ scattering based on the traditional current algebra technique alone. Our analysis shows that its S-wave component vanishes in the chiral limit. This implies that, when we consider the effective theory of Goldstone boson and vector meson, the $\rho^0\rho^0\pi^+\pi^-$ vertex may first appear proportional to the current quark mass or higher derivatives.

In order to derive the low energy behavior of V^0-π^+ ($V^0 = \rho^0$, ω or ϕ) scattering amplitude let us start, as in the case of pion-nucleon scattering [8], from the algebraic identity

$$
\begin{aligned}
&(-iq^\mu)(ik^\nu)\int d^4x e^{iq\cdot x}\int d^4y e^{-ik\cdot y}\left\langle b\,\mathbf{P}_f\,\sigma\left|T J^+_{5\mu}(x)\,J^-_{5\nu}(y)\right|a\,\mathbf{P}_i\,\lambda\right\rangle\\
&=\frac{1}{2}\int d^4x e^{iq\cdot x}\int d^4y e^{-ik\cdot y}\\
&\quad\times\Big[(-iq^\mu)\left\{\delta(x^0-y^0)\left\langle b\,\mathbf{P}_f\,\sigma\left|\left[J^-_{50}(\mathbf{y},x^0),J^+_{5\mu}(\mathbf{x},x^0)\right]\right|a\,\mathbf{P}_i\,\lambda\right\rangle\right\}\\
&\qquad+(ik^\nu)\left\{\delta(x^0-y^0)\left\langle b\,\mathbf{P}_f\,\sigma\left|\left[J^+_{50}(\mathbf{x},x^0),J^-_{5\nu}(\mathbf{y},x^0)\right]\right|a\,\mathbf{P}_i\,\lambda\right\rangle\right\}\\
&\qquad-iq^\mu\left\langle b\,\mathbf{P}_f\,\sigma\left|T J^+_{5\mu}(x)\,\partial^\nu J^-_{5\nu}(y)\right|a\,\mathbf{P}_i\,\lambda\right\rangle\\
&\qquad+ik^\nu\left\langle b\,\mathbf{P}_f\,\sigma\left|T\partial^\mu J^+_{5\mu}(x)\,J^-_{5\nu}(y)\right|a\,\mathbf{P}_i\,\lambda\right\rangle\Big],
\end{aligned}
\tag{1}
$$

where $J^{\pm}_{5\mu} = \frac{1}{\sqrt{2}}(J^1_{5\mu} \pm iJ^2_{5\mu})$, $|a\,\mathbf{P}_i\,\lambda\rangle$ denotes the vector meson states of momentum $\mathbf{P}_i$ (boldface letter means the usual three space momentum) and helicity λ. The internal label a signifies the neutral vector meson states in the nonet basis, ρ^0, ω and ϕ. Throughout this paper all the current quark masses are set equal to zero. The masses of ω, ϕ and $\rho^\pm$ are taken equal for simplicity although there is no a priori reason to expect it even in the flavor symmetric limit. The divergence of the axial current, $\partial^\mu J^{\pm}_{5\mu}(x)$, is kept in the last two terms of (1) to accommodate possible influence of chiral anomaly.

What we are interested in here is whether the S-wave component of V^0-π^+ scattering amplitude is of order 1 or not in the simultaneous limit of $k^\mu, q^\mu \to 0$. Since the pions are massless in the chiral-symmetric limit, the above limit is restated more precisely as the $\epsilon \to 0$ limit where

$$\begin{aligned} q^\mu &= (\epsilon, \epsilon \mathbf{n}_q), \\ k^\mu &= (\epsilon, \epsilon \mathbf{n}_k), \qquad |\mathbf{n}_q| = |\mathbf{n}_k| = 1. \end{aligned} \tag{2}$$

Presumably the dominant contribution from the first two terms on the RHS of Eq. (1) is

$$\begin{aligned} \frac{1}{2}\Big[-iq^\mu \Big\langle b\,\mathbf{P}_f\,\sigma \Big| \int d^4x \left[Q_5^-(x^0), J_{5\,\mu}^+(\mathbf{x}, x^0) \right] \Big| a\,\mathbf{P}_i\,\lambda \Big\rangle \\ + ik^\nu \Big\langle b\,\mathbf{P}_f\,\sigma \Big| \int d^4y \left[Q_5^+(x^0), J_{5\,\nu}^-(\mathbf{y}, y^0) \right] \Big| a\,\mathbf{P}_i\,\lambda \Big\rangle \Big], \end{aligned} \tag{3}$$

which arises from the leading term of the expansion $e^{iq\cdot x}e^{-ik\cdot y} = 1 + iq\cdot x - ik\cdot y + \cdots$. The axial current $J_{5\,\mu}^\pm(x)$ transforms under the axial charge $Q_5^\mp$ to the third component of iso-vector current $J_\mu^3(x)$:

$$\begin{aligned} \left[Q_5^-(x_0), J_{5\,\mu}^+(\mathbf{x}, x^0) \right] &= -J_\mu^3(\mathbf{x}, x^0), \\ \left[Q_5^+(x_0), J_{5\,\mu}^-(\mathbf{x}, x^0) \right] &= J_\mu^3(\mathbf{x}, x^0). \end{aligned} \tag{4}$$

Thus Eq. (1) reduces, to the leading order in ϵ, to

$$\begin{aligned} &\lim_{\epsilon\to 0} \int d^4x e^{iq\cdot x} \int d^4y e^{-ik\cdot y}\; q^\mu k^\nu \left\langle b\,\mathbf{P}_f\,\sigma \left| T J_{5\,\mu}^+(x)\, J_{5\,\nu}^-(y) \right| a\,\mathbf{P}_i\,\lambda \right\rangle \\ &= -\frac{1}{2}(q+k)^\mu \int d^4x \left\langle b\,\mathbf{P}_f\,\sigma \left| J_\mu^3(x) \right| a\,\mathbf{P}_i\,\lambda \right\rangle \\ &\quad + \frac{1}{2}\Big[-iq^\mu \int d^4x e^{iq\cdot x} \int d^4y e^{-ik\cdot y} \left\langle b\,\mathbf{P}_f\,\sigma \left| T J_{5\,\mu}^+(x) \partial^\nu J_{5\,\nu}^-(y) \right| a\,\mathbf{P}_i\,\lambda \right\rangle \\ &\qquad + ik^\nu \int d^4x e^{iq\cdot x} \int d^4y e^{-ik\cdot y} \left\langle b\,\mathbf{P}_f\,\sigma \left| T \partial^\mu J_{5\,\mu}^+(x) J_{5\,\nu}^-(y) \right| a\,\mathbf{P}_i\,\lambda \right\rangle \Big]. \end{aligned} \tag{5}$$

Let us first examine the LHS of eq. (5). Recalling that the state vectors describe neutral vector meson states, the singular components with respect to q, k in

$$\left\langle b\,\mathbf{P}_f\,\sigma \left| T J_{5\,\mu}^+(x)\, J_{5\,\nu}^-(y) \right| a\,\mathbf{P}_i\,\lambda \right\rangle \tag{6}$$

can be classified into three categories depicted in Figs. 2-4, analogous to the case of nucleon-pion scattering. The contribution of Fig. 2 gives rise to the V^0-π^+ scattering amplitude

$$\mathcal{A}(\mathrm{Fig.}\,2) = -f_\pi^2 \mathcal{M}\left(V^a(P_i,\lambda) + \pi^+(k) \to V^b(P_f,\sigma) + \pi^+(q)\right) \times i(2\pi)^4 \delta^4(P_f + q - P_i - k). \tag{7}$$

The diagrams of Figs. 3 and 4 are singular if and only if the intermediate states are degenerate in mass with the external vector meson. Recalling the realistic hadronic spectrum, we may assume that the non-anomalous parts of the current operator in these diagrams do not induce a singular contribution: For instance, axial-vector meson (such as A_1) is much heavier than ρ, ω, or ϕ. Thus the only possible source of singular contribution is the anomalous part of the currents with the propagation of the charged ρ meson as the intermediate state. Isospin invariance (or conservation of G-parity) guarantees that this contribution vanishes for the ρ^0-π^+ scattering. In the other cases, each diagram multiplied by a factor $q^\mu k^\nu$ may give contribution of order ϵ at most to the LHS of Eq. (5). (See eq. (2) for the definition of ϵ.) Actually the S-wave component of the sum of diagrams in Figs. 3(a) and 3(d), Figs. 3(b) and 3(c), are found to be of order ϵ^2 respectively. The demonstration is quite similar as in the case of pion-nucleon scattering. Likewise one can see that the cancellation of order one terms occurs between the diagrams in Fig. 4 .

Similar consideration persists as well for the last two terms on the RHS of Eq. (5), due to the absence of Goldstone-pole contribution in this case (no such a pole in $\partial^\mu J^\pm_{5\mu}$), and they vanish in the limit $\epsilon \to 0$.

Finally, since the vector current J^3_μ transforms as $\mathcal{C} J^3_\mu \mathcal{C}^\dagger = -J^3_\mu$ under charge conjugation, the first term of the RHS, which is the matrix element of J^3_μ between two V^0 states, vanishes identically.

Thus we conclude that the V^0-π^+ scattering amplitude must vanish in the limit of $\epsilon \to 0$.

Various models of vector meson proposed thus far will help to check to the general argument [9].

Since the case of HLS approach has been explored before [5], we do not repeat it again.

A. Massive Vector Yang-Mills Approach

First we take up the massive Yang-Mills approach incorporating vector meson only [10,11]. Following the procedure of Ref. [11], the vector meson of nonet V_μ is introduced from A_μ^L and A_μ^R which transform under the local chiral transformation $(g_L, g_R) \in G \equiv U(3)_L \times U(3)_R$

$$
\begin{aligned}
A_\mu^{L\,\prime} &= g_L A_\mu^L g_L^\dagger + \frac{i}{g} g_L \partial_\mu g_L^\dagger, \\
A_\mu^{R\,\prime} &= g_R A_\mu^R g_R^\dagger + \frac{i}{g} g_R \partial_\mu g_R^\dagger,
\end{aligned} \tag{8}
$$

in such a way that

$$
\begin{aligned}
A_\mu^L &= \xi(\pi)\rho_\mu \xi(\pi)^\dagger + \frac{i}{g}\xi(\pi)\partial_\mu \xi(\pi)^\dagger, \\
A_\mu^R &= \xi(\pi)^\dagger \rho_\mu \xi(\pi) + \frac{i}{g}\xi(\pi)^\dagger \partial_\mu \xi(\pi),
\end{aligned} \tag{9}
$$

with the Goldstone boson matrix

$$
\begin{aligned}
\xi(\pi) &= \exp\left(i\frac{\pi}{f_\pi}\right), \\
\pi &= \pi^a T^a.
\end{aligned} \tag{10}
$$

(The group generator T^a is normalized as $2\,\mathrm{Tr}(T^a\,T^b) = \delta^{ab}$ throughout the paper.) The transformation law of V_μ is non-linear [10]

$$
V_\mu' = h(\pi, g_L, g_R) V_\mu h(\pi, g_L, g_R)^\dagger + h(\pi, g_L, g_R)\partial_\mu h(\pi, g_L, g_R)^\dagger, \tag{11}
$$

as is verified from

$$
\xi(\pi') = g_L \xi(\pi) h(\pi, g_L, g_R)^\dagger = h(\pi, g_L, g_R)\xi(\pi) g_R^\dagger, \tag{12}
$$

where $h(\pi, g_L, g_R)$ is an element of the vector group $\mathrm{U(3)_V}$. If we introduce the external gauge fields $\mathcal{R}_\mu$, $\mathcal{L}_\mu$ which transform as

$$
\begin{aligned}
\mathcal{L}_\mu' &= g_L \mathcal{L}_\mu g_L^\dagger + i g_L \partial_\mu g_L^\dagger, \\
\mathcal{R}_\mu' &= g_R \mathcal{R}_\mu g_R^\dagger + i g_R \partial_\mu g_R^\dagger,
\end{aligned} \tag{13}
$$

the local $\mathrm{U(3)_L \times U(3)_R}$ invariant terms bilinear at most in V_μ with no further derivatives are

$$m_0^2 \,\mathrm{Tr}\left[(A_\mu^R - \frac{1}{g}\mathcal{R}_\mu)^2 + (A_\mu^L - \frac{1}{g}\mathcal{L}_\mu)^2\right] - B\,\mathrm{Tr}\left[(A_\mu^L - \frac{1}{g}\mathcal{L}_\mu)U(A^{R\mu} - \frac{1}{g}\mathcal{R}^\mu)U^\dagger\right], \tag{14}$$

where $U \equiv \xi^2$, plus the Yang-Mills term. (In general the terms like

$$\left\{\mathrm{Tr}(A_\mu^R - \frac{1}{g}\mathcal{R}_\mu)\right\}^2 + \left\{\mathrm{Tr}(A_\mu^L - \frac{1}{g}\mathcal{L}_\mu)\right\}^2, \tag{15}$$

are also permitted. However the conclusion will not be affected by the presence of those terms so that these kinds of terms will not be mentioned explicitly. The same remark should also be recalled hereafter.) From eq. (9), eq. (14) contains two V' s only in the form of the vector meson mass term. This is the same situation as in the HLS approach; the $V^0V^0\pi^+\pi^-$ vertex appears with explicit symmetry violation by quark mass or with higher derivatives, e.g.,

$$\begin{aligned}&\mathrm{Tr}\left[\left(A_\mu^L - \frac{1}{g}\mathcal{L}_\mu\right)\mathcal{M}\left(A^{R\mu} - \frac{1}{g}\mathcal{R}^\mu\right)U\right]\\ &+\mathrm{Tr}\left[\left(A_\mu^L - \frac{1}{g}\mathcal{L}_\mu\right)U\left(A^{R\mu} - \frac{1}{g}\mathcal{R}^\mu\right)\mathcal{M}^\dagger\right],\end{aligned} \tag{16}$$

where the current quark mass matrix $\mathcal{M}$ is considered to transform as

$$\mathcal{M} \to g_L \mathcal{M} g_R^\dagger, \tag{17}$$

when the quark mass term appears as $-\bar{q}_L\mathcal{M}q_R$ in QCD Langrangian. (Hereafter those terms as in (16) will be called as higher-order terms in derivative expansion.)

B. Generalized Hidden Local Symmetry Approach

The generalized HLS approach incorporates the vector and axial-vector mesons as the gauge fields of HLS $[\mathrm{U(3)_L \times U(3)_R}]_{\mathrm{local}}$ [6]. Since its description is rather cumbersome, the reader should consult with Ref. [6]. Eqs. (7.63) and (7.64) of Ref. [6] are the terms which involve at most two vector or axial-vector mesons of lowest order in derivatives. In the unitary gauge of HLS, these become

$$\mathcal{L} = af_\pi^2 \,\mathrm{Tr}\left[\left(gV_\mu - \frac{i}{2f_\pi^2}[\pi, \partial_\mu \pi] + \cdots\right)^2\right]$$
$$+(b+c)f_\pi^2 \,\mathrm{Tr}\left[\left(gA_\mu - \frac{b}{b+c}\partial_\mu \pi + \cdots\right)^2\right] + \text{kinetic terms}, \tag{18}$$

where V_μ and A_μ are the vector and axial-vector nonets, and a, b and c are the constants to be determined from the experiments. The axial hidden gauge transformation

$$(V_\mu + A_\mu) \to (V'_\mu + A'_\mu) = \frac{i}{g}\,\eta\,\partial_\mu \eta^\dagger + \eta\,(V_\mu + A_\mu)\,\eta^\dagger,$$
$$(V_\mu - A_\mu) \to (V'_\mu - A'_\mu) = \frac{i}{g}\,\eta^\dagger\,\partial_\mu \eta + \eta^\dagger\,(V_\mu - A_\mu)\,\eta, \tag{19}$$

where

$$\eta = \exp\left(i\frac{b}{b+c}\frac{\pi}{f_\pi}\right), \tag{20}$$

rotates away the transition term between A_μ and π in eq.(18). This manipulation induces the $\rho^0\rho^0\pi^+\pi^-$ interaction

$$\frac{1}{f_\pi^2}(M_{A_1}^2 - M_V^2)\left(\frac{b}{b+c}\right)^2 \rho_\mu^0 \rho^{0\,\mu}\pi^+\pi^-, \tag{21}$$

with $M_{A_1}^2 = (b+c)f_\pi^2 g^2$ and $M_V^2 = af_\pi^2 g^2$, to the Lagrangian. However the $\pi^+\rho^0 A_1^-$ interaction also appears at the same time;

$$\frac{i}{f_\pi^2}(M_{A_1}^2 - M_V^2)\,\frac{b}{b+c}\,\left(A_1^{-\,\mu}\rho_\mu^0\pi^+ - A_1^{+\,\mu}\rho_\mu^0\pi^-\right). \tag{22}$$

As a result the invariant amplitude of ρ^0-π^+ scattering becomes

$$\begin{aligned}
&\mathcal{M}\left(\rho^0(P_i,\lambda) + \pi^+(k) \to \rho^0(P_f,\sigma) + \pi^+(q)\right)\\
&= \varepsilon_\mu^{(\sigma)\,*}(\mathbf{P}_f)\varepsilon_\nu^{(\lambda)}(\mathbf{P}_i)\left[\frac{2(M_{A_1}^2 - M_V^2)}{f_\pi^2}\left(\frac{b}{b+c}\right)^2 g_{\mu\nu}\right.\\
&\quad + \frac{(M_{A_1}^2 - M_V^2)^2}{f_\pi^2}\left(\frac{b}{b+c}\right)^2\left\{\frac{1}{(P_i+k)^2 - M_{A_1}^2}\left(g^{\mu\nu} - \frac{(P_i+k)^\mu(P_i+k)^\nu}{M_{A_1}^2}\right)\right.\\
&\qquad\qquad \left.\left. + \frac{1}{(P_f-k)^2 - M_{A_1}^2}\left(g^{\mu\nu} - \frac{(P_f-k)^\mu(P_f-k)^\nu}{M_{A_1}^2}\right)\right\}\right].
\end{aligned} \tag{23}$$

It is an easy algebraic task to observe that the S-wave component of this amplitude is actually $\mathcal{O}(\epsilon^2)$ in the limit (2) , consistent with the low energy theorem shown in the above general discussion.

C. Antisymmetric Tensor Field Approach

The vector and axial-vector mesons can be incorporated into the effective theory as antisymmetric tensor fields [12]. Then there does not arise a mixing between axial-vector mesons and Goldstone bosons.

The nonet $R_{\mu\nu}$ ($R = V$, or A) transforms under G as

$$R'_{\mu\nu} = h(\pi, g_L, g_R) R_{\mu\nu} h(\pi, g_L, g_R)^\dagger, \tag{24}$$

by $h(\pi, g_L, g_R)$ defined in eq. (12). The possible R^2 terms with the least numbers of derivatives and quark mass which include pion fields are, e.g.,

$$\begin{aligned} &\mathrm{Tr}\,\left[\left(\xi^\dagger \mathcal{M} \xi^\dagger + \xi \mathcal{M}^\dagger \xi\right) R_{\mu\nu} R^{\mu\nu}\right], \\ &\mathrm{Tr}\,[R_{\mu\alpha} u^\alpha R_{\mu\beta} u_\beta], \\ &\mathrm{Tr}\,\left[A_{\mu\nu}\left[\left(\xi^\dagger \mathcal{M} \xi^\dagger - \xi \mathcal{M}^\dagger \xi\right), V^{\mu\nu}\right]\right], \end{aligned} \tag{25}$$

where

$$\begin{aligned} u_\mu &= i\xi^\dagger (D_\mu U) \xi^\dagger, \\ D_\mu U &= \partial_\mu U + iU\mathcal{R}_\mu - i\mathcal{L}_\mu U. \end{aligned} \tag{26}$$

Contrary to the generalized HLS approach, both $\rho^0\rho^0\pi^+\pi^-$ and $\pi^+\rho^0 a_1^-$ vertices do not exist in the chiral limit. Such a respect is similar to the one seen in the massive Yang-Mills approach to incorporate vector and axial-vector mesons. (See eq.(7.105) of Ref. [6].)

To summarize, current algebra analysis predicts that the amplitude of ρ^0-π^+ scattering shall vanish in the low pion momentum. We have checked this fact based on the various chiral-symmetric models of vector meson. The effective Lagrangian of Ref. [4] are not written

to include the vector meson field explicitly. Thus the problem is whether the structure found in their formula for the hadronic light-by-light scattering amplitude can be obtained starting from some chiral-symmetric model of vector meson. The result here seems to indicate that it is impossible in the chiral limit.

The effective Lagrangian of Ref. [4] may be used to describe the effects to be induced by integrating out some resonance (other than rho meson) if it scatters with pion of order one in the low energy limit. But setting M_X to the mass (≥ 1 [GeV]) of such a resonance will show that the corresponding contribution to the muon $g-2$ is negligible compared to the "rho-meson" contribution based on the HLS Lagrangian [5], a chiral-symmetric model of the vector meson.

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FIGURES

FIG. 1. Hadronic light-by-light scattering effect on muon $g-2$. The solid and dashed lines represent muon and photon respectively. The blob part corresponds to the light-by-light scattering amplitude by hadron(s).

FIG. 2. V^0-π^+ scattering contribution. Here the dotted and bold lines denote the charged pion and the vector meson respectively. The blob part corresponds to the amplitude of pion and vector meson scattering.

FIG. 3. Singular components with one pion pole. The intermediate state here is the charged vector meson. The dark blob corresponds to the contact term of the axial current operator. The shaded blob denotes the anomalous $V^0V^-\pi^+$ (or $V^0V^+\pi^-$) vertex.

FIG. 4. Singular components in which two axial currents are directly attached to the vector meson line.

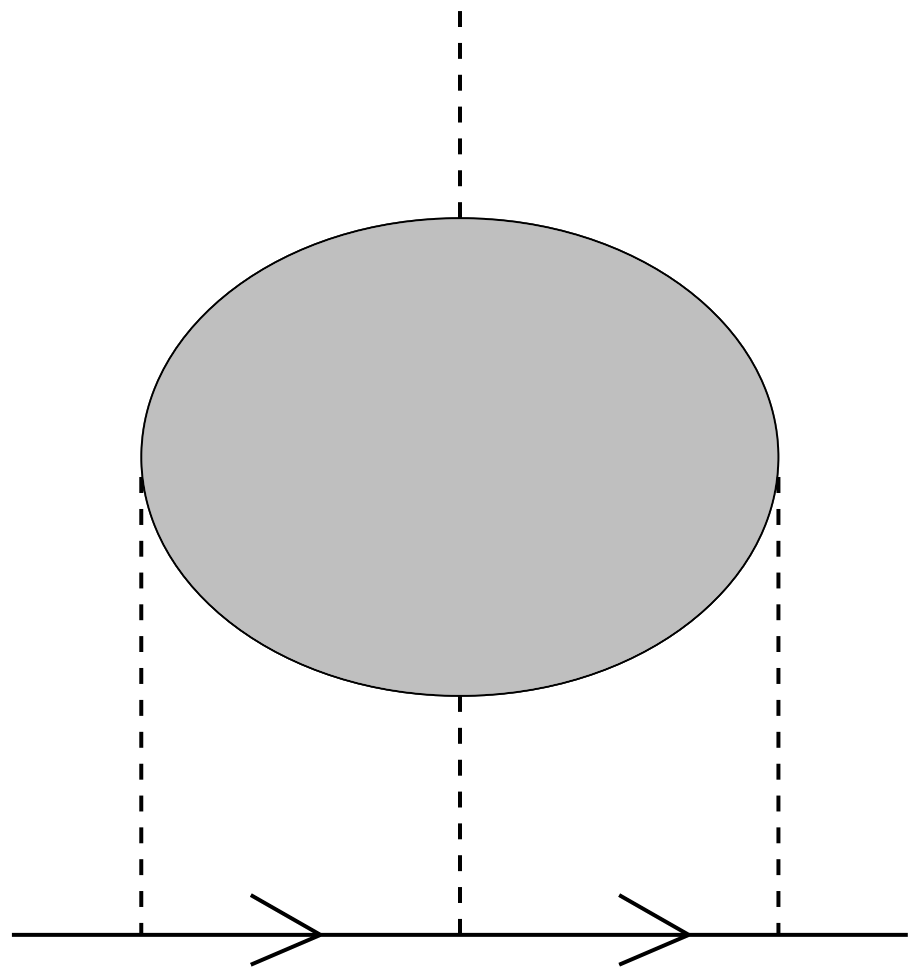

FIG. 1

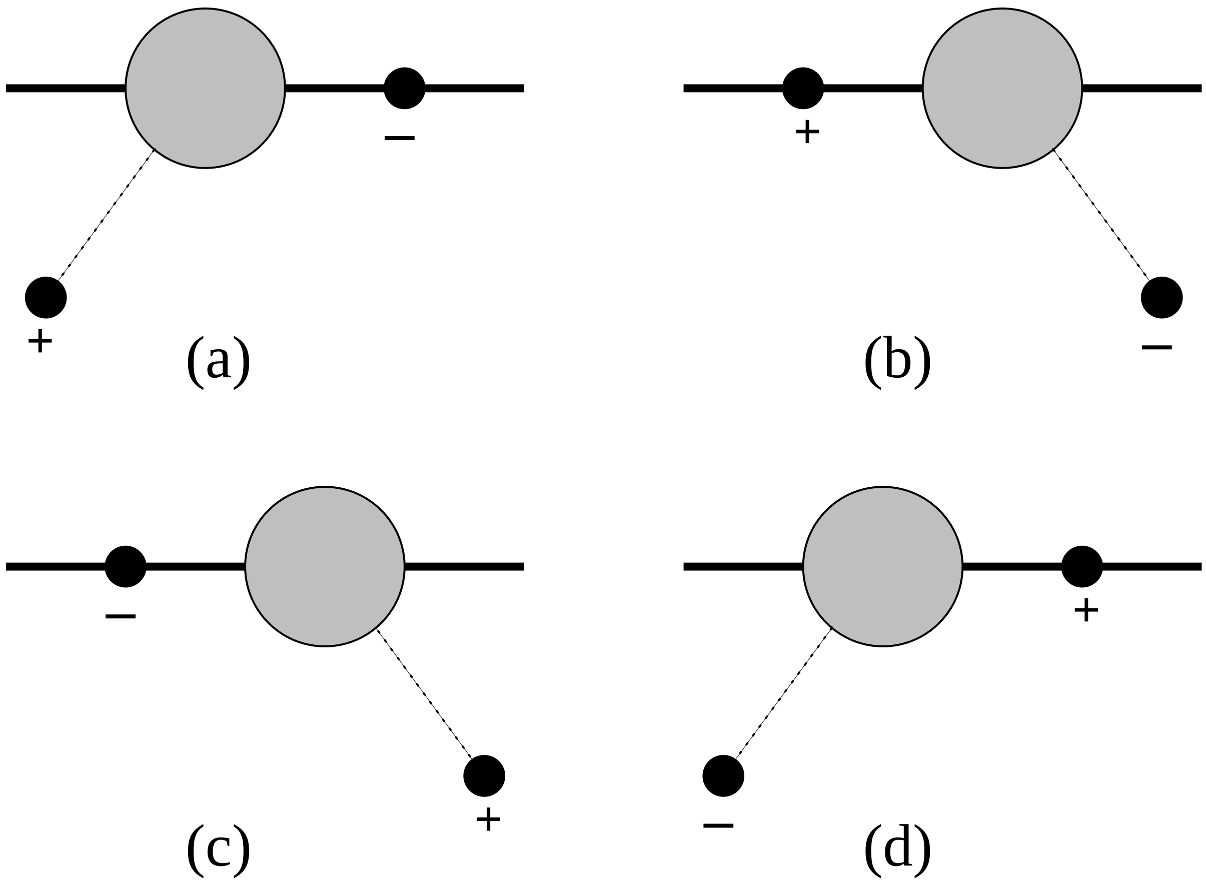

FIG. 3

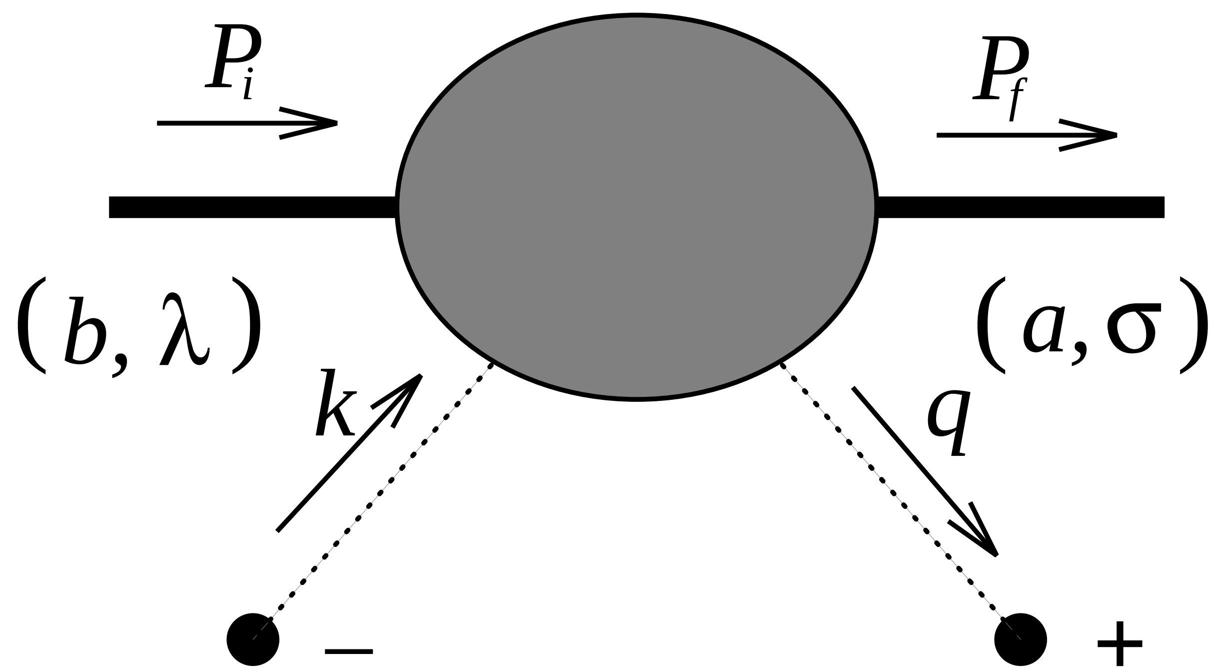

FIG. 2

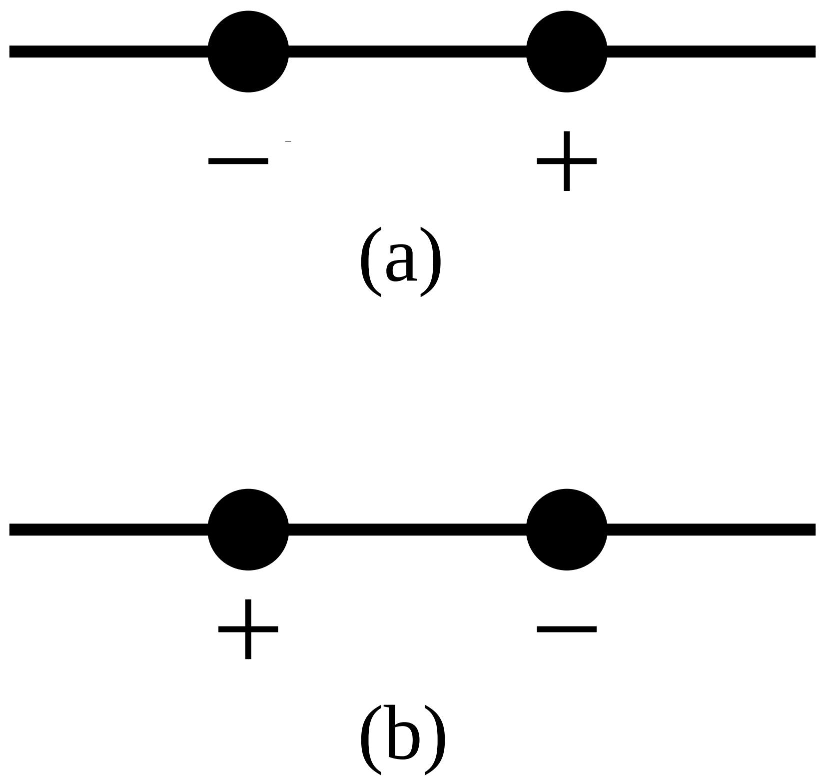

FIG. 4